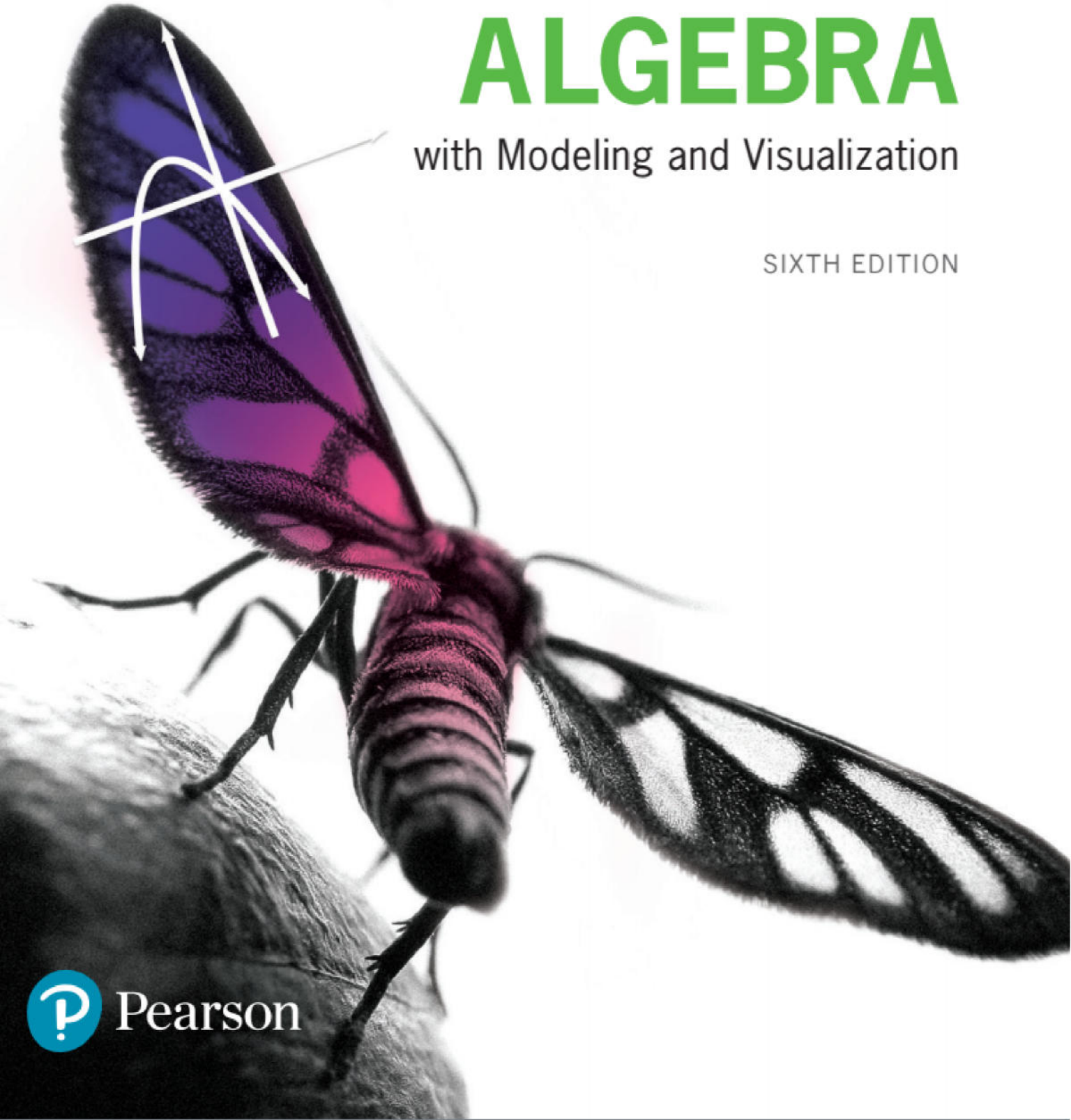
ROCKSWOLD

COLLEGE ALGEBRA

with Modeling and Visualization

SIXTH EDITION



Pearson

6th edition

College Algebra

with Modeling & Visualization

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and
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Pearson

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To my father, Palmer, who celebrated one century of living,
and to my granddaughter, Nora, who celebrated one year of living,
the same month.

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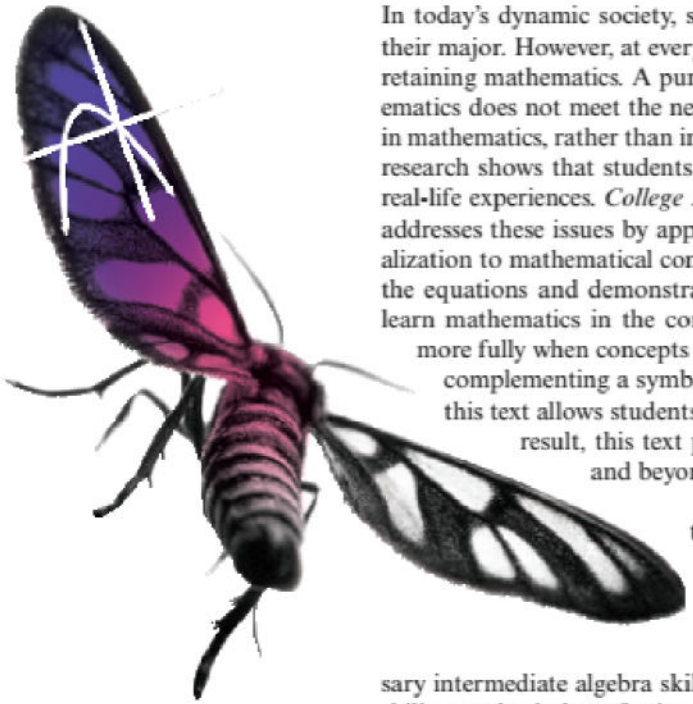
Contents

Foreword



In today's dynamic society, students need to understand mathematics regardless of their major. However, at every level, students continue to have difficulty learning and retaining mathematics. A purely traditional, or abstract, approach to teaching mathematics does not meet the needs of most of our students and has lead to exclusivity in mathematics, rather than inclusivity. In order to both learn and retain mathematics, research shows that students must see a connection between the concepts and their real-life experiences. *College Algebra with Modeling and Visualization,* Sixth Edition, addresses these issues by appropriately connecting applications, modeling, and visualization to mathematical concepts and skills. This text consistently gives meaning to the equations and demonstrates that mathematics *is* relevant. It allows students to learn mathematics in the context of their experiences. Students learn mathematics more fully when concepts are presented not only symbolically but also visually. By complementing a symbolic approach with an emphasis on visual presentations, this text allows students to absorb information faster and more intuitively. As a result, this text promotes inclusivity and diversity within our discipline and beyond.

The concept of a function is the unifying theme in this text with an emphasis on the rule of four (verbal, graphical, numerical, and symbolic representations). A flexible approach allows instructors to strike their own balance of skills, rule of four, applications, modeling, and technology. Rather than reviewing all of the necessary intermediate algebra skills in the first chapter, this text integrates required math skills seamlessly by referring students "just in time" to Chapter R, "Basic Concepts from Algebra and Geometry." Instructors are free to assign supplemental homework from this chapter. Students also have additional opportunities to review their skills in the MyMathLab® course when needed. Here, personalized homework and quizzes are readily available on a wide variety of review topics.

Students frequently do not realize that mathematics is transforming our society. To communicate this fact, the author has established a website at www.garyrockswold.net. Here, several resources are available, including a number of invited addresses given by the author. These presentations are accessible to students and allow them to understand the big picture of how mathematics influences everyone's life.

Gary Rockswold

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Preface

Changes to the Sixth Edition

The Sixth Edition continues to gives meaning to the numbers that students encounter by developing concepts in context through the use of applications, multiple representations, and visualization. Seamlessly integrated real-life connections, graphs, tables, and meaningful data help deepen student understanding. To this end, there are many new and exciting changes to the Sixth Edition.

- All *See the Concept* boxes now have a corresponding video that takes a student through the concept step-by-step to make them more accessible for students. Additional *See the Concept* boxes have also been included.
- Hundreds of application examples and exercises have been updated to bring timely meaning and relevance to the mathematics.
- Over 600 new exercises have been added throughout the text, both at the basic and higher levels of difficulty.
- Clarity has been emphasized throughout to make the text easier for students to read with the use of bubbles, labels, and headings.
- At the request of reviewers, the definition of intercept has been changed to be a point rather than a real number.
- More emphasis on domain and range in context has been included.
- More critical thinking about graphical interpretation has been added. These examples and exercises often ask students to identify characteristics of a graph, such as intercepts, zeros, extrema, and intervals where the graph is increasing or decreasing.
- **Chapter 1** includes the new topics of finding percent change and the center of a circle by completing the square. Interval notation is introduced earlier in Chapter 1. More discussion of graphing linear functions by hand, interpreting domain and range in context, applying the Pythagorean theorem, and determining an appropriate calculator window has been added.
- **Chapter 2** has additional examples and exercises covering piecewise-defined functions, absolute value inequalities, and critical thinking about graphs of functions. A new subsection on percentages has been included and a more complete discussion of the x-intercept method has also been added.
- **Chapter 3** now has a graphical derivation of the vertex formula that is accessible to students. There is additional emphasis on domain and range in context and also identifying the domain and range of translated and reflected functions.
- **Chapter 4** has a new subsection covering graphs of power functions having integer exponents. Much of Section 4.2 has been rewritten to make it more accessible for students.
- **Chapter 5** has a new subsection covering exponential and logarithmic inequalities. More discussion of linear and exponential growth, simplifying functions and their domains, and logarithmic and exponential forms has been added. More modeling examples and exercises that require students to select a modeling function have been added to Section 5.7.
- **Chapter 6** has new coverage of supply and demand applications along with finding equilibrium prices and quantities. Additional business and social network applications have also been included. A new discussion of steps for solving a system of equations using the elimination method has been added.
- **Chapter 7** has new examples and exercises for finding the standard equation of a circle by completing the square.

- **Chapter 8** has new *See the Concept* boxes that help explain the distinction between arithmetic and geometric sequences.
- **Appendix A** is new and contains several Collaborative Activities that can be completed in or out of class. These activities, or projects, are application-based, include discussion of results, and often require connections with previous concepts.

Features

The Sixth Edition places an emphasis on conceptual learning, developing students' understanding of The Big Picture, and providing more tools for classrooms looking to incorporate more activities and group projects.

NEW!

- **Collaborative Activities**

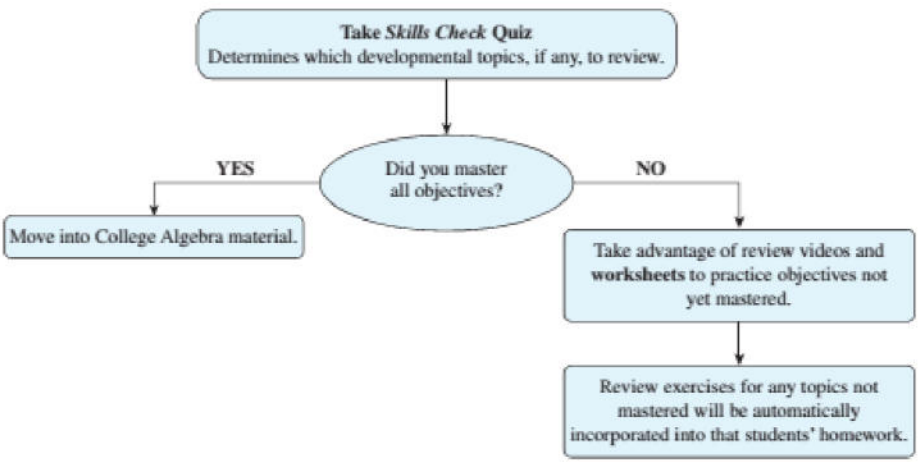
  A NEW set of 8 collaborative activities is available in **Appendix A**. Developed by the author, these activities follow a project-based learning approach and allow students to actively explore real-world challenges and apply what they know to produce results that matter. Some activities focus on a specific concept while others span multiple concepts, requiring students to synthesize their knowledge and leading to a greater understanding of The Big Picture. Even more projects can be found in the new *Guided Notebook*, available in **MyMathLab** or bundled with the book.

NEW!

- **Guided Notebook with Integrated Review Worksheets**

  This NEW student supplement, authored by Laura J. Younts (*Santa Fe College*), offers a one-stop-shop for student support and engagement. Each section contains a structured lecture outline that begins with an application that students fill in during lecture, followed by a group activity to complete in class. A student reflection section gives students the opportunity to record questions they have for the next class, as well as a space for "my homework," where students write down the work to be done at home. An extended project is included for most chapters. This notebook is ideal for any classroom looking to incorporate more active learning.

  Also included in the *Guided Notebook* are the Rockswold Integrated Review worksheets. These worksheets offer additional practice exercises of relevant intermediate algebra topics with ample space for students to show their work. The **MyMathLab Integrated Review** course option is just like our time-tested **MyMathLab** course but with even more resources specially designed to bring underprepared students up to speed for college algebra:



NEW! MyMathLab®

See the Concept with Videos and Assessment

This exciting feature allows students to make important connections by walking them through detailed visualizations. Students use graphs, tables, and diagrams to learn new concepts in a concise and efficient way. NEW for the Sixth Edition, this popular feature is brought to life with videos, created for concepts in the text where students would benefit from *seeing* the math explained visually. They are integrated throughout the eText for immediate access when students need it most and are also available in the multimedia library for easy in-class use. Additionally, each *See the Concept* video has an accompanying **MyMathLab** assessment question, making these videos truly assignable (see pages 38, 54, 108, and 125). Look for them in the **MyMathLab** Assignment Manager with the label "STC."

NEW! MyMathLab®

Getting Started with "Set Up & Solve" Questions

This feature occurs in select examples that require multistep solutions. *Getting Started* helps students develop an overall problem-solving strategy before they begin writing a detailed solution. NEW for the Sixth Edition, these problems have been rendered in **MyMathLab** utilizing a **Set Up & Solve** technique. These multipart exercises require students to show the setup of the solution for a particular exercise as well as the solution to gauge a students' conceptual understanding of the topic (see pages 7 and 80). Look for them in the **MyMathLab** Assignment Manager with the label "Set Up & Solve."

NEW! MyMathLab®

Putting It All Together

This unique and helpful feature at the end of every section summarizes techniques and reinforces the mathematical concepts presented in the section. It is given in an easy-to-follow grid. NEW for the Sixth Edition, these questions are rendered in **MyMathLab** as conceptual questions, asking students to classify, sort, categorize, or order mathematical expressions, graphs, and terms (see pages 113 and 368). Look for them in the **MyMathLab** Assignment Manager with the label "PIAT."

Checking Basic Concepts

This feature, included after every two sections, provides a small set of exercises that can be used as mixed review. These exercises require about 15 to 20 minutes to complete and can be used for collaborative, peer-to-peer learning during class (see pages 119 and 152).

Critical Thinking

This feature, included in most sections, poses a question that requires students to take a concept a step further. They can be used for either classroom discussion or homework (see pages 55 and 181).

Now Try

This feature occurs after each example. It suggests a similar exercise students can work to see if they understand the concept presented in the example (see pages 19 and 85).

Making Connections

This feature, which occurs throughout the text, shows students how concepts covered previously are related to new concepts being presented (see pages 155 and 240).

Comment Boxes

This feature allows graphs, tables, and symbolic explanations to be labeled in such a way that a concept is easier to understand. The explanation is now tied closely to a graph, table, or equation (see pages 31 and 99).

Algebra and Geometry Review Notes

Throughout the text, Algebra and Geometry Review Notes, located in the margins, direct students "just in time" to Chapter R, where important topics in algebra

and geometry are reviewed. Instructors can use this chapter for extra review or refer students to it as needed. The feature *frees* instructors from having to frequently review materials from intermediate algebra and geometry (see pages 111 and 185). In addition, quizzes and personalized homework on review skills are available in **MyMathLab**—see "Getting Ready" assignments.

- **Chapter Summary Grids**

  Chapter summaries are presented in an easy-to-read grid format, listing the Concept and providing an Explanation and Example. They allow students to quickly review key concepts from the chapter (see pages 256 and 375).

Exercise Sets

The exercise sets are the heart of any mathematics text, and this text includes a large variety of instructive exercises. Each set of exercises covers skill building, mathematical concepts, and applications. Graphical interpretation and tables of data are often used to extend students' understanding of mathematical concepts. The exercise sets are graded carefully and categorized according to topic, making it easy for an instructor to select appropriate assignments. Additional exercise sets include **Chapter Review Exercises, Extended and Discovery Exercises, Cumulative Review Exercises, and Writing About Mathematics.** For the Sixth Edition, three new categories of exercises are available:

NEW!

- **Critical Thinking**

  These exercises ask students to take a mathematical concept a step further than what is discussed in the text. They challenge students to think beyond the pages of the book (see pages 67–68 and 94–95).

NEW!

- **Interpret & Analyze in Context**

  Identified by a green gear icon, these exercises indicate where students need to interpret or analyze math used to describe real life (see pages 96–97 and 147).

NEW!

- **Checking Symbolic Skills**

  These exercises provide a preview into important topics that students will see again in calculus (see pages 47, 314, and 353–354).

Data-Driven Revision

A goal of this revision is to improve learning outcomes for students. To help achieve this goal, we analyzed aggregated student usage and performance data from the previous edition's **MyMathLab** course. The results of this analysis yielded specific improvements to this edition, including:

- **Adjusted difficulty levels**—We analyzed the easiest and most challenging exercises in the text to see whether adjustments needed to be made to those exercises or to the instruction in the text that supports them. This also allowed us to refine the progression of difficulty in the exercise sets so that they unfold evenly from simpler to more challenging.
- **Added or adjusted content**—We analyzed exercise usage data to determine where content might need to be added to this text and its **MyMathLab** course. We also analyzed exercise and eText usage data to help inform whether content that was seldom used might be covered more succinctly.

Get the Most Out of MyMathLab®

MyMathLab is the leading online homework, tutorial, and assessment program for teaching and learning mathematics, built around Pearson's best-selling content. MyMathLab helps students and instructors improve results; it provides engaging experiences and personalized learning for each student so learning can happen in any environment. Plus, it offers flexible and time-saving course management features to allow instructors to easily manage their classes while remaining in complete control, regardless of course format.

Preparedness

One of the biggest challenges in many mathematics courses is making sure students are adequately prepared with the prerequisite skills needed to successfully complete their coursework. MyMathLab offers a variety of content and course options to support students with just-in-time remediation and key-concept review.

- **MyMathLab with Integrated Review**—available for Developmental Mathematics through Calculus—can be used for just-in-time prerequisite review or corequisite courses. These courses provide videos on review topics, along with premade, assignable skills-check quizzes and personalized review homework assignments.
- In recent years many new course models have emerged as institutions "redesign" to help improve retention and results. At Pearson, we're focused on creating solutions tailored to support your plans and programs. In addition to the new **Integrated Review** courses, we offer non-STEM pathways and STEM-track options.

Used by more than 37 million students worldwide, MyMathLab delivers consistent, measurable gains in student learning outcomes, retention, and subsequent course success.

www.mymathlab.com

Resources for Success

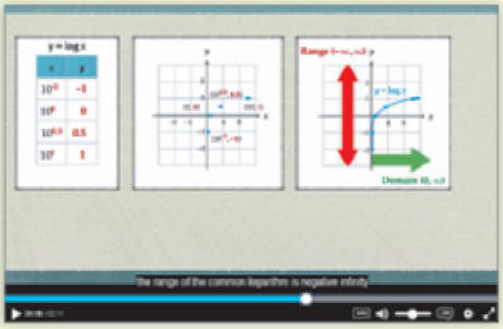
MyMathLab® Online Course for *College Algebra with Modeling & Visualization*, by Rockswold

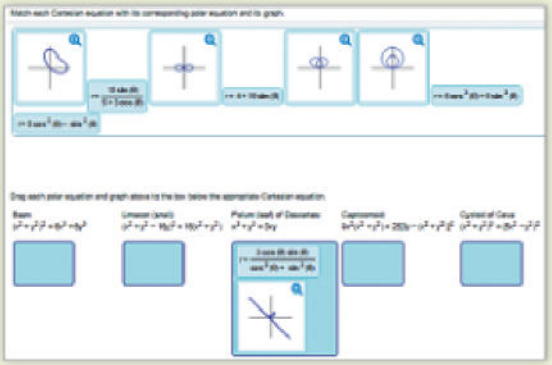
(access code required)

MyMathLab is available to accompany Pearson's market leading text offerings. To give students a consistent tone, voice, and teaching methods, each text's flavor and approach is tightly integrated throughout the accompanying MyMathLab course, making learning the material as seamless as possible.

NEW! See the Concept Videos with Assessment

See the Concept videos support visualization and conceptual understanding. Many students do better if they can visualize the math. These videos have been created for concepts in the text where students would benefit from seeing the math worked out. They are integrated throughout the eText and also available in the multimedia library. All videos have an accompanying **MyMathLab** assessment question, making these videos truly assignable.





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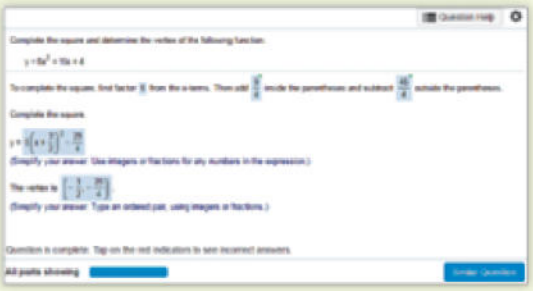

www.mymathlab.com

Resources for Success

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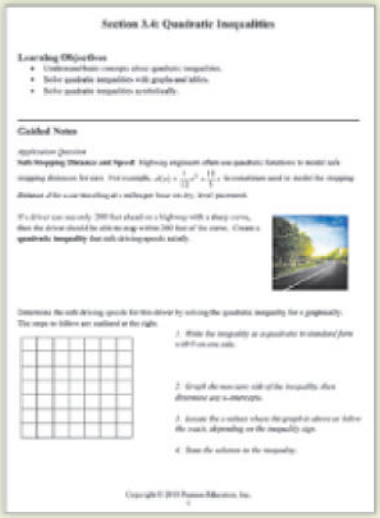
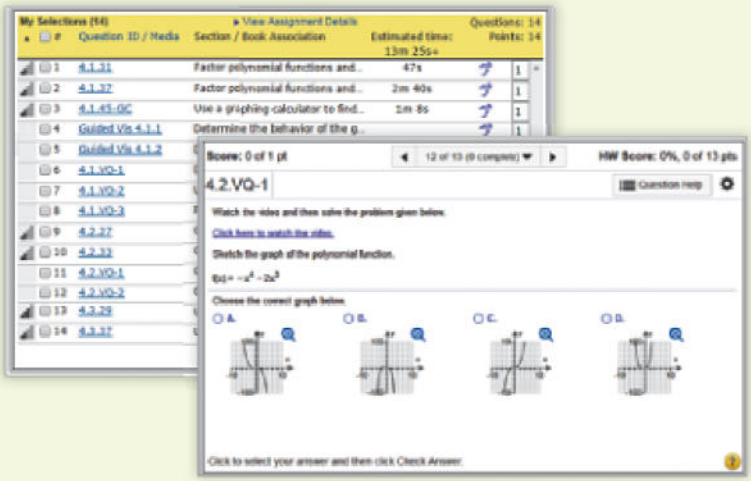
(access code required)

NEW! Guided Notebook with Integrated Review Worksheets

An invaluable companion to the book, the *Guided Notebook*, by Laura J. Younts (*Santa Fe College*) contains structured lecture outlines, additional application questions, group activities, reflection sections, and Extended Projects. **Integrated Review** worksheets for extra practice on intermediate algebra topics are included. This notebook is ideal for any classroom looking to incorporate more active learning.



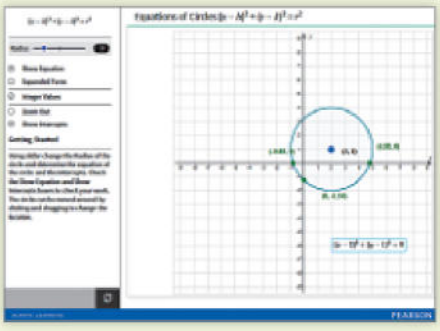


NEW! Enhanced Sample Assignments

These newly redesigned assignments put a powerful combination of author expertise and dynamic **MyMathLab** content at your fingertips. Composed of author-selected exercises and our newest question types, including video assessment and interactive figures with assessment, these fully editable, prebuilt assignments give you the best of the best for each section in a snap!

NEW! Guided Visualizations

These engaging interactive figures bring mathematical concepts to life, helping students visualize the concepts through directed explorations and purposeful manipulation. Excellent to use during lecture, *Guided Visualizations* are also assignable in **MyMathLab** with accompanying assessment.



www.mymathlab.com

Resources for Success

Instructor Resources

Available from **www.pearsonhighered.com** or from within your MyMathLab course.

Annotated Instructor's Edition

The Instructor's Edition includes all answers to the exercise sets. *Teaching Examples* provide an extra set of examples for instructors to present in class, doubling the number of examples available for instructors, and *Teaching Tips* offer helpful ideas about presenting topics or teaching from the text. Solutions and PowerPoint® slides are available for *Teaching Examples*.

Instructor's Solutions Manual (download only)

This resource provides complete solutions to all text exercises, excluding Writing about Mathematics.

Guided Solutions for Collaborative Activities (download only)

This resource provides objectives, a list of concepts covered, and the solutions for each Collaborative Activity found in Appendix A. Objectives and concepts for the extended projects found in the Guided Notebook are also available here.

Instructor's Testing Manual (download only)

Written by David Atwood (*Rochester Community and Technical College*), this resource provides prepared tests for each chapter of the text as well as answers.

PowerPoint® Lecture Slides (download only)

Written and designed specifically for this text, these lecture slides provide an outline for presenting definitions, figures, and key examples from the text.

TestGen® (download only)

TestGen® (www.pearsoned.com/testgen) enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text.

Student Resources

Additional resources to help student success.

Guided Notebook with Integrated Review Worksheets

This new student supplement, authored by Laura J. Younts (*Santa Fe College*), offers a one-stop-shop for student support and engagement. Each section contains a structured lecture outline that begins with an application that students fill in during lecture, followed by a group activity to complete in class. A student reflection section gives students the opportunity to record questions they have for the next class, as well as a space for "my homework," where students write down the work to be done at home. An extended project is included for most chapters.

Also included in the Guided Notebook are the Rockswold *Integrated Review Worksheets*, offering additional practice exercises for relevant intermediate algebra topics.

Editable files are available for download within MyMathLab, or a hard copy can be bundled with the book or MyMathLab access card.

Student's Solutions Manual

This resource provides complete solutions to all odd-numbered text exercises, excluding Writing about Mathematics and Extended and Discovery Exercises. Available within MML or as a hard copy.

Acknowledgments

Many individuals contributed to the development of this textbook. I would like to thank the following reviewers, whose comments and suggestions were invaluable in preparing this edition of the text.

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Thanks go to Wendy Rockswold, who gave invaluable assistance and encouragement throughout the project. She also supplied several of the photographs found throughout the text.

A special thank you goes to the many students and instructors who used the first five editions of this text. Their suggestions were insightful. Please feel free to contact me at either *gary.rockswold@mnsu.edu* or www.garyrockswold.net with your comments. Your opinion is important.

Gary Rockswold

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1 Introduction to Functions and Graphs

Have you ever thought about how we "live by the numbers?" Money, sports, digital televisions, speed limits, grade point averages, gas mileages, and temperatures are all based on numbers. When we are told what our weight, blood pressure, body mass index, and cholesterol levels are, it can even affect how we feel about ourselves. Numbers permeate our society.

Numbers are an essential part of mathematics. Mathematics is used not only in science and technology; it is also used to describe almost every facet of life, including consumer behavior, social networks, and the Internet. Most vocations and professions require a higher level of mathematical understanding than in the past. Seldom are the mathematical expectations for employees lowered, as the workplace becomes *more* technical—not less. Mathematics gives people the reasoning skills to solve problems from work and life.

In this chapter we discuss numbers and how functions are used to do calculations with these numbers. Understanding numbers and mathematical concepts is essential to understanding and dealing with the many changes that will occur in our lifetimes. Mathematics makes life easier!

1.1 Numbers, Data, and Problem Solving

- Recognize common sets of numbers
- Evaluate expressions by applying the order of operations
- Learn scientific notation and use it in applications
- Apply problem-solving strategies
- Calculate percent change

Introduction

Because society is becoming more complex and diverse, our need for mathematics is increasing dramatically each year. Numbers are essential to our everyday lives. For example, the iPhone 7 has either a 4.7- or 5.5-inch display, 3 gigabytes of RAM, and a 12-megapixal rear camera. It can operate at temperatures between 32° and 95°F. (**Source:** Apple Corporation.)

Mathematics not only provides numbers to describe new products but also gives us problem-solving strategies. This section discusses basic sets of numbers and introduces some essential problem-solving strategies.

Sets of Numbers

One important set of numbers is the set of **natural numbers**. This set comprises the *counting numbers* $N = \{1, 2, 3, 4, \dots\}$. Another important set of numbers is the **whole numbers** $W = \{0, 1, 2, 3, \dots\}$. Whole numbers include the natural numbers and the number 0.

The **integers** $I = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$ are a set of numbers that contains the natural numbers, their additive inverses (negatives), and 0.

A **rational number** can be expressed as the *ratio* of two integers $\frac{p}{q}$, where $q \neq 0$. A rational number results when an integer is divided by a nonzero integer. Thus rational numbers include fractions and the integers.

Examples of Rational Numbers

$$\frac{2}{1}, \frac{1}{3}, -\frac{1}{4}, \frac{-50}{2}, \frac{22}{7}, 0, \sqrt{25}, 1.2$$

Note that 0 and 1.2 are both rational numbers. They can be represented by the fractions $\frac{0}{1}$ and $\frac{12}{10}$. Because two fractions that look different can be equivalent, such as $\frac{1}{2}$ and $\frac{2}{4}$, rational numbers have more than one form. A rational number can always be expressed in a decimal form that either *repeats* or *terminates*. For example, $\frac{2}{3} = 0.\overline{6}$, a repeating decimal, and $\frac{1}{4} = 0.25$, a terminating decimal. The overbar indicates that $0.\overline{6} = 0.6666666\ldots$.

CRITICAL THINKING

The number 0 was invented well after the natural numbers. Many societies did not have a zero—for example, there is no Roman numeral for 0. Discuss some possible reasons for this.

Real numbers can be represented by decimal numbers. Since every rational number has a decimal form, real numbers include rational numbers. However, some real numbers cannot be expressed as a ratio of two integers. These numbers are called **irrational numbers**. The numbers $\sqrt{2}$, $\sqrt{15}$, and π are examples of irrational numbers. They can be represented by nonrepeating, nonterminating decimals.

NOTE For any positive integer a, if $\sqrt{a}$ is not an integer, then the real number $\sqrt{a}$ is an irrational number.

Real numbers are either rational or irrational numbers and can always be *approximated* by a terminating decimal.

Examples of Real Numbers

Approximately equal

$$2, -10, -131.3337, \frac{1}{3} = 0.\overline{3}, -\sqrt{5} \approx -2.2361, \sqrt{11} \approx 3.3166$$

NOTE The symbol $\approx$ means **approximately equal**. This symbol is used in place of an equals sign whenever two unequal quantities are close in value. For example, $\frac{1}{2} = 0.5$, whereas $\frac{1}{3} \approx 0.3333$.

FIGURE 1.1 illustrates how the different sets of numbers are related.

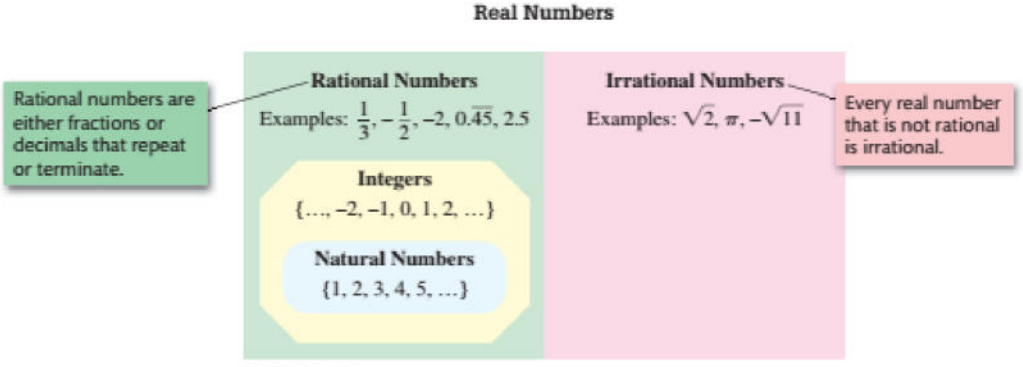

FIGURE 1.1

EXAMPLE 1 Classifying numbers

Classify each real number as one or more of the following: natural number, whole number, integer, rational number, or irrational number.

$$5, -1.2, \frac{13}{7}, -\sqrt{7}, -12, \sqrt{16}, 0$$

SOLUTION

5: natural number, whole number, integer, and rational number

-1.2: rational number

$\frac{13}{7}$: rational number

$-\sqrt{7}$: irrational number

-12: integer and rational number

$\sqrt{16} = 4$: natural number, whole number, integer, and rational number

0: whole number, integer, and rational number

Now Try Exercise 7

Order of Operations

Does $6 - 3 \cdot 2$ equal 0 or 6? Does -5^2 equal 25 or -25? **FIGURE 1.2** correctly shows that $6 - 3 \cdot 2 = 0$ and that $-5^2 = -25$. Because multiplication is performed before subtraction, $6 - 3 \cdot 2 = 6 - 6 = 0$. Similarly, because exponents are evaluated before performing negation or multiplication, $-5^2 = (-1)5^2 = -25$. It is essential that algebraic expressions be evaluated consistently, so the following rules have been established.

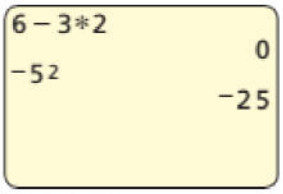

FIGURE 1.2

ORDER OF OPERATIONS

Using the following order of operations, perform all calculations within parentheses, square roots, and absolute value bars and above and below fraction bars. Then use the same order of operations to perform any remaining calculations.

1. Evaluate all exponents. Then do any negation *after* evaluating exponents.
2. Do all multiplication and division from *left to right*.
3. Do all addition and subtraction from *left to right*.

NOTE Negation can generally be thought of as multiplying by -1. Because exponents are evaluated before multiplication, exponents are also evaluated before negation.

EXAMPLE 2 Evaluating arithmetic expressions by hand

Evaluate each expression by hand.

(a) $3(1-5)^2-4^2$ **(b)** $\dfrac{10-6}{5-3}-4-|7-2|$

SOLUTION

(a) $3(1-5)^2-4^2 = 3(-4)^2-4^2$
$= 3(16)-16$
$= 48-16$
$= 32$

(b) $\dfrac{10-6}{5-3}-4-|7-2| = \dfrac{4}{2}-4-|5|$
$= 2-4-5$
$= -2-5$
$= -7$

NOTE $(-4)^2 = (-4)(-4) = 16$ and $-4^2 = (-1)4^2 = -(4)(4) = -16$.

Now Try Exercises 19 and 21

Scientific Notation

Numbers that are large or small in absolute value are often expressed in scientific notation. **TABLE 1.1** lists examples of numbers in **standard (decimal) form** and in **scientific notation**.

Applications of Scientific Notation

Standard Form	Scientific Notation	Application
93,000,000 mi	9.3×10^7 mi	Distance to the sun
205,000	2.05×10^5	Number of U.S. cell towers in 2015
9,000,000,000	9×10^9	Estimated world population in 2050
0.00000538 sec	5.38×10^{-6} sec	Time for light to travel 1 mile
0.000005 cm	5×10^{-6} cm	Size of a typical virus

TABLE 1.1

EXAMPLE 3 Writing a number in scientific notation

Write 0.000578 in scientific notation.

SOLUTION To write 0.000578 in scientific notation, start by moving the decimal point to the right of the first nonzero digit, 5, to obtain 5.78.

Decimal Form		Scientific Notation	
$0.0005.78$	$\rightarrow$	5.78×10^{-4}	Move the decimal point right.
$1\ 2\ 3\ 4$			

Since the decimal point was moved four places to the *right*, the exponent of 10 is *negative* 4, or -4. If the decimal point had been moved to the *left*, the exponent of 10 would be *positive* 4.

Now Try Exercise 35

Here is a formal definition of scientific notation.

SCIENTIFIC NOTATION

A real number r is in **scientific notation** when r is written as $c \times 10^n$, where $1 \le |c| < 10$ and n is an integer.

An Application The next example demonstrates how scientific notation appears in the description of a new technology.

EXAMPLE 4 Analyzing the energy produced by your body

Nanotechnology is a technology of the very small—on the order of one billionth of a meter. Researchers are using nanotechnology to power tiny devices with energy from the human body. (**Source:** Z. Wang, "Self-Powered Nanotech," *Scientific American*.)

(a) Write one billionth in scientific notation.

(b) While typing, a person's fingers generate about 2.2×10^{-3} watt of electrical energy. Write this number in standard (decimal) form.

SOLUTION

(a) One billionth can be written as $\frac{1}{1{,}000{,}000{,}000} = \frac{1}{10^9} = 1 \times 10^{-9}$.

(b) Move the decimal point in 2.2 three places to the left: $2.2 \times 10^{-3} = 0.0022$.

Now Try Exercise 89

The next example illustrates how to evaluate expressions in scientific notation.

EXAMPLE 5 Evaluating expressions by hand

Evaluate each expression. Write your result in scientific notation and standard form.

(a) $(3 \times 10^3)(2 \times 10^4)$ **(b)** $(5 \times 10^{-3})(6 \times 10^5)$ **(c)** $\frac{4.6 \times 10^{-1}}{2 \times 10^2}$

SOLUTION

(a)
$$\begin{aligned}(3 \times 10^3)(2 \times 10^4) &= 3 \times 2 \times 10^3 \times 10^4 && \text{Commutative property}\\ &= 6 \times 10^{3+4} && \text{Add exponents.}\\ &= 6 \times 10^7 && \text{Scientific notation}\\ &= 60{,}000{,}000 && \text{Standard form}\end{aligned}$$

(b)
$$\begin{aligned}(5 \times 10^{-3})(6 \times 10^5) &= 5 \times 6 \times 10^{-3} \times 10^5 && \text{Commutative property}\\ &= 30 \times 10^2 && \text{Add exponents.}\\ &= 3 \times 10^3 && \text{Scientific notation}\\ &= 3000 && \text{Standard form}\end{aligned}$$

(c)
$$\begin{aligned}\frac{4.6 \times 10^{-1}}{2 \times 10^2} &= \frac{4.6}{2} \times \frac{10^{-1}}{10^2} && \text{Multiplication of fractions}\\ &= 2.3 \times 10^{-1-2} && \text{Subtract exponents.}\\ &= 2.3 \times 10^{-3} && \text{Scientific notation}\\ &= 0.0023 && \text{Standard form}\end{aligned}$$

Now Try Exercises 53, 55 and 57

Algebra Review
To review exponents, see Chapter R (page R-8).

Calculators Calculators often use E to express powers of 10. For example, 4.2×10^{-3} might be displayed as 4.2E−3. On some calculators, numbers can be entered in scientific notation with the EE key.

EXAMPLE 6 Computing in scientific notation with a calculator

Approximate each expression. Write your answer in scientific notation.

(a) $\left(\frac{6 \times 10^3}{4 \times 10^6}\right)(1.2 \times 10^2)$ **(b)** $\sqrt{4500\pi}\left(\frac{103 + 450}{0.233}\right)^3$

SOLUTION

(a) The given expression is entered in two ways in **FIGURE 1.3**. Note that in both cases

$$\left(\frac{6 \times 10^3}{4 \times 10^6}\right)(1.2 \times 10^2) = 0.18 = 1.8 \times 10^{-1}.$$

(b) Be sure to insert parentheses around 4500π and around the numerator, $103 + 450$, in the expression $\sqrt{4500\pi}\left(\frac{103 + 450}{0.233}\right)^3$. From **FIGURE 1.4** we can see that the result is approximately 1.59×10^{12}.

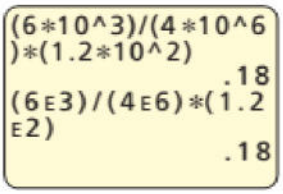

FIGURE 1.3

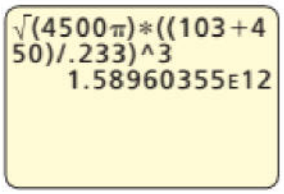

FIGURE 1.4

Now Try Exercises 61 and 63

EXAMPLE 7 Computing with a calculator

Use a calculator to evaluate each expression. Round answers to the nearest thousandth.

(a) $\sqrt[3]{131}$ **(b)** $\pi^3 + 1.2^2$ **(c)** $\frac{1 + \sqrt{2}}{3.7 + 9.8}$ **(d)** $|\sqrt{3} - 6|$

SOLUTION

(a) On some calculators the cube root can be found by using the MATH menu. If your calculator does not have a cube root key, enter 131^(1/3). From the first two lines in **FIGURE 1.5**, we see that $\sqrt[3]{131} \approx 5.079$.

(b) Do *not* use 3.14 for the value of π. Instead, use the built-in key to obtain a more accurate value of π. From the bottom two lines in **FIGURE 1.5**, $\pi^3 + 1.2^2 \approx 32.446$.

(c) When evaluating this expression be sure to include parentheses around the numerator and around the denominator. Most calculators have a special square root key that can be used to evaluate $\sqrt{2}$. From the first three lines in **FIGURE 1.6**, $\frac{1 + \sqrt{2}}{3.7 + 9.8} \approx 0.179$.

(d) The absolute value can be found on some calculators by using the MATH NUM menus. From the bottom two lines in **FIGURE 1.6**, $|\sqrt{3} - 6| \approx 4.268$.

Algebra Review
To review cube roots, see Chapter R (Page R-39).

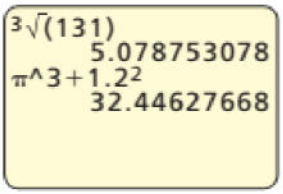

FIGURE 1.5

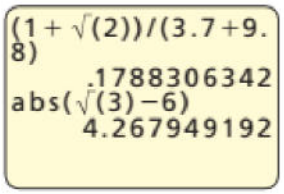

FIGURE 1.6

Now Try Exercises 67, 69, 71, and 73

Problem Solving

Many problem-solving strategies are used in algebra. However, in this subsection we focus on two important strategies that are used frequently: making a sketch and applying one or more formulas. These strategies are illustrated in the next three examples.

EXAMPLE 8 Finding the speed of Earth

Earth travels around the sun in an approximately circular orbit with an average radius of 93 million miles. If Earth takes 1 year, or about 365 days, to complete one orbit, estimate the orbital speed of Earth in miles per hour.

SOLUTION

Getting Started Speed S equals distance D divided by time T, $S = \frac{D}{T}$. We need to find the number of miles Earth travels in 1 year and then divide it by the number of hours in 1 year. ▶

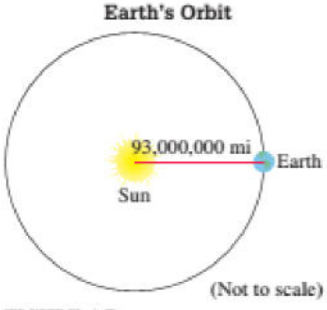

FIGURE 1.7

Distance Traveled Make a sketch of Earth orbiting the sun, as shown in FIGURE 1.7. In 1 year Earth travels the circumference of a circle with a radius of 93 million miles. The circumference of a circle is $2\pi r$, where r is the radius, so the distance D is

$$D = 2\pi r = 2\pi(93{,}000{,}000) \approx 584{,}300{,}000 \text{ miles.}$$

Hours in 1 Year The number of hours T in 1 year, or 365 days, equals

$$T = 365 \times 24 = 8760 \text{ hours.}$$

Speed of Earth $S = \frac{D}{T} = \frac{584{,}300{,}000}{8760} \approx 66{,}700$ miles per hour.

Now Try Exercise 91

Geometry Review
To find the circumference of a circle, see Chapter R (Page R-2).

Many times in geometry we evaluate formulas to determine quantities, such as perimeter, area, and volume. In the next example we use a formula to determine the number of fluid ounces in a soda can.

EXAMPLE 9 Finding the volume of a soda can

FIGURE 1.8

The volume V of the cylindrical soda can in FIGURE 1.8 is given by $V = \pi r^2 h$, where r is its radius and h is its height.

(a) If $r = 1.4$ inches and $h = 5$ inches, find the volume of the can in cubic inches.

(b) Could this can hold 16 fluid ounces? (*Hint:* 1 cubic inch equals 0.55 fluid ounce.)

SOLUTION

(a) $V = \pi r^2 h = \pi(1.4)^2(5) = 9.8\pi \approx 30.8$ cubic inches.

(b) To find the number of fluid ounces, multiply the number of cubic inches by 0.55.

$$30.8 \times 0.55 = 16.94$$

Yes, the can could hold 16 fluid ounces.

Now Try Exercise 97

Percent Change The Consumer Price Index (CPI) is often referred to as the "cost of living index" and is the numerical scale most commonly used to measure inflation. It tracks the prices of basic consumer goods. If the CPI changes from c_1 to c_2, then the **percent change** is given by

$$\frac{c_2 - c_1}{c_1} \times 100.$$